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PSYC214: Statistics
Lecture 2 – One factor between-participants ANOVA – Part I

Michaelmas Term,
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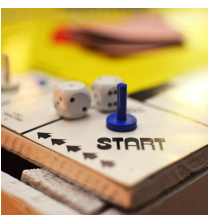
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Agenda/Content for Lecture 2

- Introduction to analysis of variance (ANOVA)
- Introduction to one factor between-participants design
- Sources of variability in data
- Calculating within-group and between-group variances
- Degrees of Freedom
- Producing the F-statistic



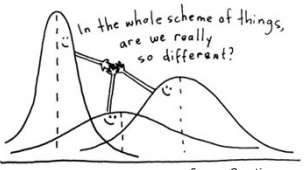
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Why conduct an analysis of variance?

- Compares means and variance
- Allows analysis of group differences for more than two groups
- Several means without inflating Type I error rate



Source: Questionpro



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Dissertation!

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ANOVA is a good weapon of choice!



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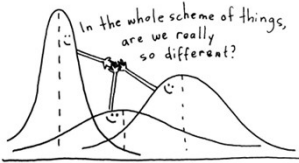
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Introduction to analysis of variance

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What do you need for a one factor between participants ANOVA?

- Three or more separate groups
- ONE categorical independent variable (i.e., one factor)
- One continuous dependent variable (outcome measure)



Source: Questionpro


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Sources of variability in data

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1. Treatment effects
2. Individual differences
3. Random (residual) errors



Within-group variability?



Between-group variability?


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Sources of variability in data

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1. **Treatment effects**
2. Individual differences
3. Random (residual) errors



Within-group variability?



Between-group variability?


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Treatment effects

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- The effects of the independent variable
- This is what we want!
- We want people who are treated differently because of our intervention to behave differently



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Sources of variability in data

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1. Treatment effects
2. **Individual differences**
3. Random (residual) errors


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Individual differences

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- Some individuals may be more proficient in memory recall
- Maybe some individuals have experience of similar tasks
- Some may have ignored instructions or had lower attention spans / motivation
- A control group can employ their own strategy, increasing the variability



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Sources of variability in data

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1. Treatment effects
2. Individual differences
3. **Random (residual) errors**

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Random (residual) errors

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- Ideally a participant would have a 'true level' at which they perform, which can always be measured accurately

1. Varying external conditions – e.g., temperature, time of day
2. State of participant (e.g. tired?)
3. Experimenter's ability to measure accurately...




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...Experimenter effects

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- Experimenters need to minimise these, so not to obscure the treatment effect
- Spread data away from the true means – i.e., increase variability and standard errors
- Reduce confidence in our estimates and a randomly plucked sample




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Within- and between- group variability


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Within-group variability
The extent to which participants within a single group or population differ, despite receiving the same treatment



Within-group variability?

Between-group variability
The extent to which overall groups differ from one another (hopefully because of our treatment)



Between-group variability? 14

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Within- and between- group variability

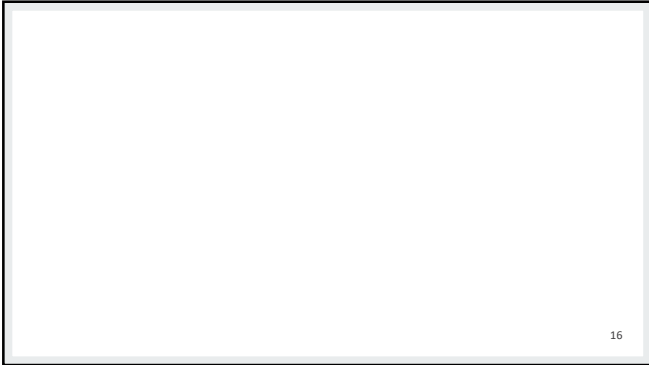
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High between-group variability No within group-variability				No between-group variability High within-group variability				Moderate between-group variability Moderate within-group variability			
Group A	Group B	Group C		Group A	Group B	Group C		Group A	Group B	Group C	
10	20	30		10	15	5		10	10	20	
10	20	30		25	20	25		10	20	20	
10	20	30		30	30	25		10	20	30	
10	20	30		35	40	45		20	20	30	
10	20	30		50	45	50		20	30	30	
Mean	10	20	30	Mean	30	30	30	Mean	14	20	26
S	0	0	0	S	14.6	12.8	18.0	S	5.5	7.1	5.5

Tables adapted from Roberts and Russo (1999)


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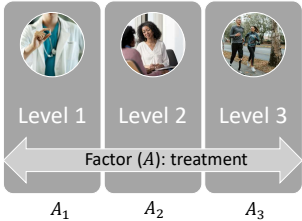
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Factors and levels (Example 1)


- Factor: **treatment**
- 3 levels
 - Medication
 - Counselling
 - Exercise



A_1 A_2 A_3

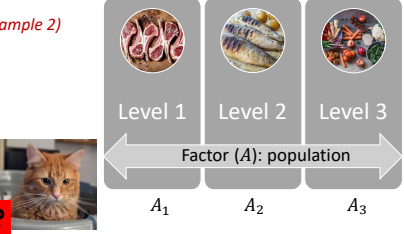
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Factors and levels (Example 2)

- Factor: **population**
- 3 levels:
 - A₁ Meat eater
 - A₂ Pescatarian
 - A₃ Vegetarian



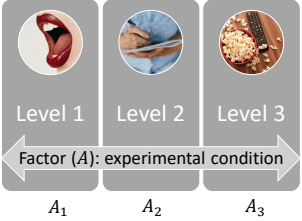
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Factors and levels (Example 3)

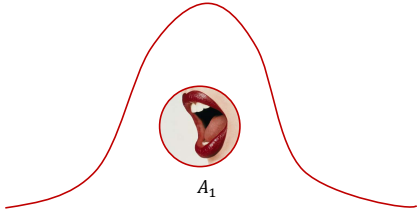
- Factor: **experimental condition**
- 3 levels:
 - A₁ Verbal negative feedback
 - A₂ Written negative feedback
 - A₃ Control (no feedback)



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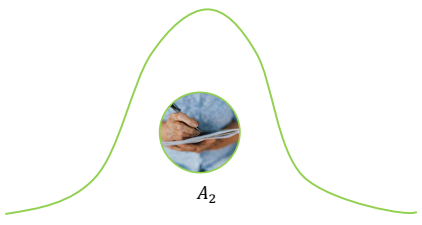


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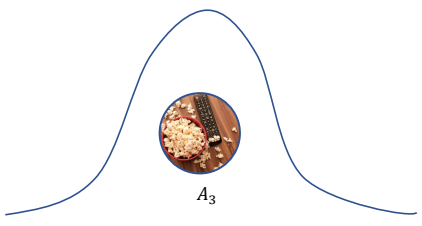
A_2

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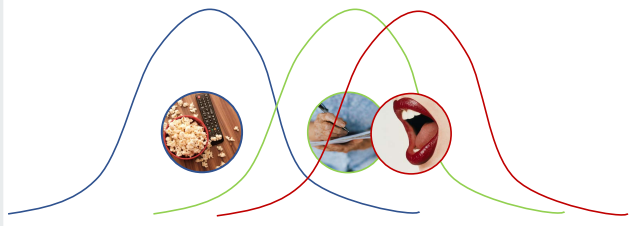
A_3

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Introduction to analysis of variance

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Testing for differences

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- **H₀ the Null Hypothesis**
- Under H₀, the samples come from the same population
- $\mu_1 = \mu_2 = \mu_3$ [No difference in the population means]
- Experimental effect = 0
- All differences are due to individual differences + random (residual) errors

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Testing for differences

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- **H₁ the Experimental Hypothesis**
- Under H₁, the samples come from the different populations.
- $\mu_1 \neq \mu_2 \neq \mu_3$ [Population means are different]
- Experimental effect \neq 0
- Differences are due to individual differences, random (residual) errors **AND** the experimental effect

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Introduction to analysis of variance

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{Signal}}{\text{Noise}}$$


$$F = \frac{\text{Signal}}{\text{Noise}}$$

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The F ratio

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

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The F ratio

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{treatment effects} + \text{individual differences} + \text{random (residual) errors}}{\text{individual differences} + \text{random (residual) errors}}$$

experimental error


experimental error

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The F ratio

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{treatment effects} + \text{individual differences} + \text{random (residual) errors}}{\text{individual differences} + \text{random (residual) errors}}$$


$$F = \frac{\text{treatment effects} + \text{experimental error}}{\text{experimental error}}$$

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Introduction to analysis of variance

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$$F = \frac{\text{Signal}}{\text{Noise}}$$

$$F = \frac{\text{Signal}}{\text{Noise}}$$


The more treatment effects are standing out away from experimental error – i.e., the larger the signal is from the noise, the larger in magnitude the F value. The larger the F, the less likely that differences in scores are caused by chance.

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Calculating between-group variance Lancaster University 


$F = \frac{\text{between-group variance}}{\text{within-group variance}}$



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Mean (\bar{A}) Lancaster University 



A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6

Total set of scores


$$\bar{X} = \frac{\sum X}{N}$$




Number of scores

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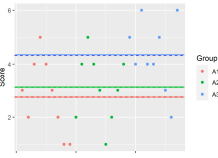
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Mean (\bar{A})




A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$






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Grand Mean (\bar{Y})



A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

$$\bar{Y} = \frac{\bar{A}_1 + \bar{A}_2 + \bar{A}_3 + \dots + \bar{A}_k}{k}$$

\bar{Y} = The grand mean of averages
k = number of levels


$$\bar{Y} = \frac{2.78 + 3.11 + 4.33}{3}$$




$$\bar{Y} = 3.41$$

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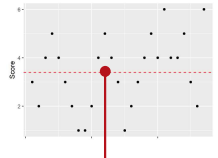
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Grand Mean (\bar{Y})



A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$



Grand mean (\bar{Y})

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Total between-group variance Lancaster University

$$\text{total between group variance} = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2 \text{ (and so on)}}{\text{total between group degrees of freedom}}$$

A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

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Total between-group variance Lancaster University

$$\text{total between group variance} = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2 \text{ (and so on)}}{\text{total between group degrees of freedom}}$$

A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

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Total between-group variance Lancaster University

$$\text{total between group variance} = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2 \text{ (and so on)}}{\text{total between group degrees of freedom}}$$

A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

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Total between-group variance Lancaster University 

$$\text{total between group variance} = \frac{9(-0.63)^2 + 9(-0.30)^2 + 9(0.92)^2}{2}$$

N_{A_1} = Number of scores for A_1
= 9

N_{A_2} = Number of scores for A_2
= 9


N_{A_3} = Number of scores for A_3
= 9

A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

$\bar{Y} = 3.41$

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Total between-group variance Lancaster University 

$$\text{total between group variance} = \frac{9(0.40) + 9(0.09) + 9(0.85)}{2}$$

N_{A_1} = Number of scores for A_1
= 9

N_{A_2} = Number of scores for A_2
= 9


N_{A_3} = Number of scores for A_3
= 9

A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

$\bar{Y} = 3.41$

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Total between-group variance Lancaster University 

$$\text{total between group variance} = \frac{3.60 + 0.81 + 7.65}{2} = 6.037 \text{ (with rounding)}$$

N_{A_1} = Number of scores for A_1
= 9

N_{A_2} = Number of scores for A_2
= 9


N_{A_3} = Number of scores for A_3
= 9

A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$

$\bar{Y} = 3.41$


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Calculating between-group variance Lancaster University 

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{6.037}{\text{within-group variance}}$$




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Up to now...

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$


$$F = \frac{6.037}{\text{within-group variance}}$$


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Calculating within-group variance


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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$


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Total within-group variance


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$$\text{total within group variance} = \frac{SS \text{ level } A_1 + SS \text{ level } A_2 + SS \text{ level } A_3 \text{ (and so on)}}{\text{total within group degrees of freedom}}$$

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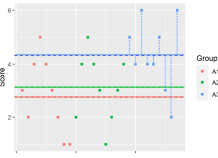
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Mean




A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$






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Total within-group variance



total within group variance = $\frac{SS \text{ level } A_1 + SS \text{ level } A_2 + SS \text{ level } A_3 \text{ (and so on)}}{\text{total within group degrees of freedom}}$

SS level A_1
= Sums of squares for level 1

SS level A_2
= Sums of squares for level 2


SS level A_3
= Sums of squares for level 3

A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$
$\bar{Y} = 3.41$		




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Total within-group variance



total within group variance = $\frac{\sum(A_1 - \bar{A}_1)^2 + (A_2 - \bar{A}_2)^2 + (A_3 - \bar{A}_3)^2 + \text{(and so on)}}{\text{total within group degrees of freedom}}$

A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$
$\bar{Y} = 3.41$		

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Total within-group variance

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total within group variance = $\frac{\sum(A_1 - 2.78)^2 + (A_2 - 3.11)^2 + (A_3 - 4.33)^2 + \dots}{\text{total within group degrees of freedom}}$

A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6

$\bar{A}_1 = 2.78$ $\bar{A}_2 = 3.11$ $\bar{A}_3 = 4.33$ $\bar{Y} = 3.41$

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Degrees of freedom

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Within-groups degrees of freedom

- For within-groups degrees of freedom, we add up the number of participants for each level - 1
- Mathematically this is expressed as:

$$= (N_{A1} - 1) + (N_{A2} - 1) + (N_{A3} - 1)$$

$$= (9 - 1) + (9 - 1) + (9 - 1)$$

= 24



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Total within-group variance

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total within group variance = $\frac{\sum(A_1 - 2.75)^2 + (A_2 - 3.11)^2 + (A_3 - 4.33)^2}{24}$

A ₁ scores	A ₂ scores	A ₃ scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6

$\bar{A}_1 = 2.78$ $\bar{A}_2 = 3.11$ $\bar{A}_3 = 4.33$ $\bar{Y} = 3.41$

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The F ratio

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{6.037}{1.769}$$

$F = 3.414, p = 0.05$, A statistically significant test result ($P \leq 0.05$)

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Lecture 2 – One factor between-participants ANOVA

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Review of Lecture 2

- What is Analysis of Variance
- What is a one-factor between-participants design
- Sources of variability in data
- Calculated within-group and between-group variances
- Degrees of Freedom
- Produced the F-statistic

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Check you understand today's lecture – repeat any parts of the lecture you need to.

Don't forget to ask any questions using the Discussion Forum on Moodle!

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